# **Estimating Design Floods with a Specified Return Period Using Bayesian Analysis**

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*Design floods for most dams and levees typically have return periods of 1:100 or less frequent. High hazard dams are designed to pass the Probable Maximum Flood (PMF), which typically has a return period of 1:10,000 or less frequent. In order to reduce epistemic uncertainties in estimated return periods for extreme floods, such as the PMF, it is important to incorporate as much hydrologic information into the frequency analysis as reasonably possible. This paper presents a Bayesian analysis framework, originally profiled by Viglione et al. (2013), for combining at-site flood data with temporal information on historic and paleofloods, spatial information on precipitation-frequency, and causal information on the flood processes. This framework is used to evaluate the flood hazard for Lookout Point Dam, which is a high priority dam located in the Willamette River Basin, upstream of Portland, Oregon. Flood frequency results are compared with those from the Expected Moments Algorithm (EMA). Both analysis methods produce similar results for typical censored data, such as historical floods; however, unlike the Bayesian analysis framework, EMA is not capable of accounting for uncertainty in paleoflood ages and magnitudes, nor is it capable of incorporating the causal rainfall-runoff information in a formal, probabilistic manner. Consequently, the Bayesian method considered herein provides higher confidence in the fitted flood frequency curves and resulting reservoir stage-frequency curves to be used in dam and levee safety risk assessments.*

*Keywords: Risk assessment, Bayesian, flood frequency, uncertainty*

## **Introduction**

Estimating the return period, or annual exceedance probability (AEP), for extreme floods such as the Probable Maximum Flood (PMF) is an important problem in hydrology for Dam and Levee Safety. For the risk assessment of dams and levees, the probability of failure is often conditional on the magnitude of the hydrologic loading. Therefore, it is imperative to ascertain credible estimates of exceedance probabilities of extreme floods that could lead to failure.

Design floods for most dams and levees have return periods of 1:100 or less frequent. High hazard dams are designed to pass the PMF, which typically has a return period of 1:10,000 or less frequent. Most projects in the United States (U.S.) have limited flood information, with many sites having less than 100 years of gauged flow data. Consequently, the greatest source of error in return period and quantile estimates in a flood frequency analysis is often inadequate data. In order to reduce epistemic uncertainties in the estimated return periods for extreme floods, such as the PMF, it is important to incorporate as much hydrologic information into the frequency analysis as reasonably possible.

This paper presents a Bayesian analysis framework, originally profiled by *Viglione et al.* (2013), for combining limited atsite flood data with temporal information on historic and paleofloods, spatial information on precipitation-frequency, and causal information on the flood processes. This framework is implemented in the Bayesian estimation and fitting software RMC-BestFit<sup>[3](#page-0-2)</sup>, which is used to evaluate the flood hazard for Lookout Point Dam, a high priority dam located in the Willamette River Basin, upstream of Portland, Oregon. Flood frequency results are compared with those from Expected Moments Algorithm (EMA), which is the currently accepted flood-frequency methodology in the U.S., described in Bulletin 17C (U.S. Geological Survey, 2018).

## **Bayesian Analysis Framework**

In Bayesian analysis, the values of the flood frequency distribution parameters  $\theta = (\theta_1, \theta_2, ..., \theta_n)$  are uncertain rather than fixed. The uncertainty in the parameters is represented by a *prior* probability distribution  $P(\theta)$ , which is established based on information available *a priori*. This prior distribution is not derived from the observed flow data  $D = (X_1, ..., X_n)$ , but instead comes from other sources that can be either subjective (e.g., expert opinion) or objective (e.g., previous statistical analyses). After the prior distributions and the observed flow data are specified, Bayes' theorem [\(Equation 1\)](#page-1-0) is

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<span id="page-0-2"></span><sup>&</sup>lt;sup>3</sup> The U.S. Army Corps of Engineers (USACE) Risk Management Center (RMC), in collaboration with the Engineer Research and Development Center (ERDC), developed the Bayesian estimation and fitting software RMC-BestFit to enhance and expedite flood hazard assessments within the Flood Risk Management, Planning, and Dam and Levee Safety communities of practice.

used to combine the *a priori* information about the parameters with the observed data, using the likelihood  $P(D|\theta)$ [\(Equation 2\)](#page-1-1).

$$
P(\theta | D) = \frac{P(D | \theta) \cdot P(\theta)}{\int P(D | \theta) \cdot P(\theta) \cdot d\theta}
$$
Equation 1

<span id="page-1-1"></span><span id="page-1-0"></span>
$$
P(D|\theta) = \prod_{i=1}^{N} f(X_i|\theta)
$$
 Equation 2

Where  $P(\theta|D)$  is the posterior probability density function (pdf) of  $\theta$ ;  $P(\theta)$  is the prior pdf of  $\theta$ ; and  $P(D|\theta)$  is the likelihood function. The posterior cumulative distribution function (cdf) of  $X$  now follows from the total probability theorem:

<span id="page-1-2"></span>
$$
F(X) = \sum_{i=1}^{n} P(\theta_i | D) \cdot F(X | \theta_i)
$$
 Equation 3

which is a probability-weighted sum of the cdfs under different posterior parameter sets  $\theta_1, \theta_2, ..., \theta_n$ . [Equation 3](#page-1-2) is equivalent to the *expected probability of exceedance* concept first presented by *Beard* (1960). *Stedinger*(1983) and *Kuczera* (1999) refer to this integral as the design flood distribution, and it is considered the optimal estimator of an exceedance probability.

In most cases, there is not a closed form solution to the denominator of [Equation 1.](#page-1-0) Therefore, Monte Carlo simulation techniques such as Markov Chain Monte Carlo (MCMC) are required. The RMC-BestFit software employs an adaptive Differential Evolution Markov Chain (DE-MC<sub>z</sub>) algorithm (ter Braak & Vrugt, 2008), which has proven to be very efficient<sup>[4](#page-1-4)</sup>. Several other MCMC algorithms have been successfully used in flood frequency analysis [ (Kuczera, 1999); (Reis & Stedinger, 2005); (Viglione, Merz, Salinas, & Bloschl, 2013)]. The Bayesian algorithm presented herein are consistent with those described in Australian Rainfall and Runoff Book 3: Peak Discharge Estimation (Kuczera & Franks, 2019).

[Figure 1](#page-1-3) illustrates the basic steps in Bayesian analysis (Meylan, 2012). The Bayesian approach offers a framework that is well suited to incorporate different sources of information, such as systematic flood records, historical floods, regional information, and other hydrologic information along with related uncertainties (Viglione, Merz, Salinas, & Bloschl, 2013). The Bayesian approach allows hydrologists to formally include their own expertise into the frequency analysis by choosing *a priori* distributions. The possibility to combine this information with the observed data is even more important because in hydrology, the samples are usually of limited size. Consequently, the Bayesian approach is more flexible and versatile than classical approaches.



<span id="page-1-3"></span>*Figure 1: Diagram Illustrating the Basic Steps in Bayesian Analysis (taken from (Meylan, 2012), which was originally taken from (Perreault, 2000))*

<span id="page-1-4"></span> <sup>4</sup> The MCMC algorithm in RMC-BestFit is parallelized so chains can be carried out simultaneously on a multi-core computer processor. Runtimes are on the order of a few seconds in most cases.

Bayesian flood frequency approaches have been presented in literature extensively in the past few decades. However, the Bayesian approach has only seen limited application in practice within the U.S., mainly because it is computationally intensive as compared to simpler classical methods, such as Bulletin 17B (U.S. Geological Survey, 1982) and Bulletin 17C (U.S. Geological Survey, 2018). With the speed of modern computers, this is no longer a concern.

## **Case Study – Lookout Point Dam**

Lookout Point Dam is a high priority dam located in the Willamette River Basin, upstream of Portland, Oregon. The Willamette River and its watershed lie in the greater Cascades Geological Province, which extends from British Columbia to northern California. The river has a watershed area of approximately 29,730 km<sup>2</sup>. The watershed runoff fluctuates dramatically with heavy precipitation in the winter months, snowmelt in the spring months, and relatively rain-free summers.

## **Frequency Analysis with Systematic Data**

Systematic data is collected at regular, prescribed intervals under a defined protocol (U.S. Geological Survey, 2018). Systematic inflow data from 1923 to 2008 was available for all of the Willamette River Basin dams. The discharge data was based on the 2010 Level Modified Streamflows dataset (Bonneville Power Agency, 2017), which reflects unregulated flow in the Columbia (Upper, Mid, and Lower), Kootenay, Pend-Oreille, Spokane, and Willamette river basins. The critical inflow duration for evaluating the flood hazard at Lookout Point dam was determined to be 3-days based on the typical duration of atmospheric rivers. All volume-frequency curves evaluated for this study assumed a Log-Pearson Type III (LPIII) distribution to be consistent with current U.S. flood frequency practice. The likelihood function for the systematic data used in the Bayesian analysis is written the same as [Equation 2:](#page-1-1)

$$
L_S(D|\theta) = \prod_{i=1}^{N_S} f(X_i|\theta)
$$
 Equation 4

Where D is the sample of annual maxima 3-day inflows systematically recorded  $X_1, ..., X_{N_s}$  ( $N_s = 84$  years for Lookout Point Dam); and  $f(\cdot)$  is the pdf of the variable X. [Figure 2](#page-2-0) shows the fit obtained by applying the Bayesian MCMC method described in the previous section.



#### Lookout Point 3-Day Volume-Frequency, with Systematic Data Only

<span id="page-2-0"></span>*Figure 2. Bayesian fit of the LPIII distribution to the systematic 3-day inflow data at Lookout Point Dam. All probability plots use a normal probability scale.*

The distribution corresponding to the posterior mode of the parameters is shown as a solid black line and labelled as "Most Likely" in the legend. The posterior mode is provided for consistency with the maximum likelihood method. The expected probability distribution, also known as the Bayesian posterior predictive distribution, is shown as black dash-dot line and labelled "Expected" in the legend. The 90% credible intervals are shown as black dashed lines labelled as "95% CI" and "5% CI", respectively. The observed annual maximum flood events are plotting using the Hirsch-Stedinger plotting positions (Hirsch & Stedinger, 1987).

[Figure 2](#page-2-0) also lists the first three moments (of log) for the LPIII distribution corresponding to the posterior mode, and the effective record length (ERL), which in this case is 84 years of systematic data. The most likely and expected values of  $Q_{1E-4}$  along with the 90% credible interval are provided as an example of a hypothetical design flood with a specified return period for a large dam. The 3-day PMF runoff volume for Lookout Point Dam is shown as a horizontal dashed line for reference.

As can be seen, there is tremendous uncertainty concerning the return period of the PMF runoff volume. There are well more than four orders of magnitude in uncertainty between the 90% credible bounds for the PMF return period. There are three orders of magnitude between the expected and most likely curves; the expected return period of the PMF is approximately 1:60,000 while the posterior mode predicts a PMF return period of approximately 1:95,000,000. Likewise, there is significant uncertainty in the 3-day flow for 1:10,000 (1E-4) quantile. This high level of uncertainty can lead to over or under-design of dam and levee safety modifications.

#### **Temporal Information Expansion**

Temporal information expansion is directed toward collecting information on the flood behaviour before (or after) the systematic data period (Viglione, Merz, Salinas, & Bloschl, 2013). For Lookout Point Dam, historical flood records were compiled using available documents and information from previous studies, which date back to 1813 and includes a total of eight historical flood events between 1861 and 1909. A paleoflood analysis was conducted to help characterize the longterm hydrologic record for the Middle Fork Willamette River and to develop an unregulated peak inflow frequency curve for Lookout Point. The study was based on evidence of geomorphic features that have not been eroded or deposited on during the past several thousand years, and evidence of a high-stage flood deposit from several hundred years ago. Hydraulic modelling was conducted to determine the peak discharge needed to inundate the specific geomorphic features of the Middle Fork Willamette River (U.S. Army Corps of Engineers, 2018a). The peak inflows for the historical and paleoflood data were converted to 3-day volumes based on a peak-to-volume ratio derived from similar large events in the systematic record. The results of the paleohydrology study indicated that a 3-day flow of approximately 9,200 cms has not been exceeded (non-exceedance bound) in the last 2,300 years. A paleoflood event took place approximately 370 years ago that produced a 3-day flow of approximately 2,980 cms (paleostage indicator (PSI) event) (U.S. Army Corps of Engineers, 2018a). [Figure 3](#page-3-0) shows a chronology plot of the systematic, historical, and paleoflood data. The non-exceedance bound (NEB) was treated as a perception threshold and is shown as a light red shaded box in the plot.



<span id="page-3-0"></span>

In the Bayesian analysis, historical perception thresholds, and the paleoflood NEB are considered left-censored data with the following likelihood function:

$$
L_L(D|\theta) = \prod_{i=1}^{N_L} {h \choose k} F(X_0|\theta)^{(h-k)}
$$
 Equation 5

Where  $X_0$  is the threshold;  $h$  is the threshold period;  $k$  is the number of events that exceeded the threshold during the period;  $\binom{h}{k}$  is the binomial coefficient; and  $F(\cdot)$  is the cdf of the variable  $X_0$ . The historical data and paleostage indicators (PSI) are treated as interval-censored data with the following likelihood function:

$$
L_I(D|\theta) = \prod_{i=1}^{N_I} [F(Y_{U_i}|\theta) - F(Y_{L_i}|\theta)]
$$
 Equation 6

Where there are  $N_l$  historical floods known to lie between upper and lower bounds. These likelihood formulations for historical data are consistent with those presented in *Stedinger & Cohn* (1986), *Kuczera* (1999), and *O'Connell et al.* (2002). The overall likelihood function accounting for the joint probability of occurrence of recent and historical flood observations as:

$$
L(D|\theta) = L_S(D|\theta) \cdot L_L(D|\theta) \cdot L_I(D|\theta)
$$
 *Equation 7*

In order to assess the "value" of including additional hydrologic information, such as historical and paleoflood data, into the flood frequency analysis, *Cohn, Lane, & Baier* (1997) propose an equation to compute the *effective record length* (ERL). For the purposes of this study, the equation was altered so that ERL is computed at multiple quantiles and then averaged as shown in [Equation 8.](#page-4-0)

<span id="page-4-0"></span>
$$
ERL = \frac{1}{n} \sum_{i=1}^{n} N_S \frac{Var\left[\widehat{X}_i | N_S\right]}{Var\left[\widehat{X}_i | N_T\right]}
$$
 Equation 8

Where ERL is the effective record length in years;  $n$  is the number of quantiles (exceedance probabilities) for which the variance is computed;  $\hat{X}_i$  is the flood magnitude for a given quantile;  $Var[\cdot]$  is the variance in flow magnitude for a given quantile and given flood record;  $N_s$  is the systematic record length; and  $N_T$  is the total record length (systematic plus the historical and paleoflood events, etc.).

[Figure 4](#page-5-0) shows the fit obtained by applying the Bayesian MCMC method with the inclusion of historical and paleoflood data. Accounting for the additional temporal information actually slightly increases the 90% credible bounds in the range of the 1:10,000 (1E-4) return period. However, the bounds are much narrower in the range of extreme flood quantiles as compared to the fit with systematic data only.

The paleoflood event was nearly double in size of any events from the systematic and historical data. This large event reveals a lot of information about the hydrology of extreme events in the watershed. Discovery of the paleoflood event has led to an increase in knowledge on the types of floods that could happen. In this sense, knowledge uncertainty (also called epistemic uncertainty) has been reduced. However, the ERL, as a measure of the value of including addition data, is not significantly increased from that of the systematic data alone. This is because the standard deviation and skew of the posterior mode have increased with the inclusion of the paleoflood data, reflecting the fact that there is higher natural variability than what was previously thought.

The uncertainty in the PMF return period has been reduced considerably with the temporal information. There is less than one order of magnitude between the expected and most likely curves; the expected return period of the PMF is approximately 1:20,000 while the posterior mode predicts a PMF return period of approximately 1:80,000, which is more three orders of magnitude more frequent than what was estimated using only the systematic data.

#### Lookout Point 3-Day Volume-Frequency, with Historical & Paleoflood Data



<span id="page-5-0"></span>*Figure 4. Bayesian fit of the LPIII distribution using systematic, historical, and paleoflood data at Lookout Point Dam.* 

#### **Spatial and Causal Information Expansion**

Spatial information expansion is based on using flood information from neighbouring catchments to improve flood frequency estimates at the site of interest. Causal information expansion analyses the generating mechanisms of floods in the catchment of interest (Merz & Bloschl, 2008). For Lookout Point Dam, spatial information was obtained by performing a regional precipitation-frequency analysis for the entire Willamette River Basin (U.S. Army Corps of Engineers, 2017). In this analysis, probability distributions were fit to the largest 3-day wintertime rainfall in the Willamette River Basin. The 3-day rainfall duration were considered critical for two reasons: Atmospheric Rivers are the prevailing meteorology of extreme rainfall events, which occur most often in the wintertime and have typical durations of 2-3 days; and the dams being studied in the basin were found to have critical inflow durations of 3-days. The effective record lengths of the basinaverage precipitation-frequency curves for the sub-watersheds within the greater Willamette River Basin vary between approximately 350 to 675 years, which indicates high confidence in the regional precipitation quantile estimates.

Causal information was developed by modelling rainfall-frequency events using a hydrologic model. The main benefit of modelling regional rainfall-frequency information is that available rainfall records are often much longer than the flood records[. Figure 5](#page-6-0) shows the 3-day volume-frequency curve based on the paleoflood data compared to the expected 3-day basin-average regional precipitation-frequency curve (converted to 3-day flow), shown in blue. Rainfall-frequency events were routed through the calibrated HEC-HMS model (U.S. Army Corps of Engineers, 2016b) by scaling three historical storm events: December 1963, February 1996, and November 1996. The rainfall-runoff results are shown as colored diamonds i[n Figure 5.](#page-6-0)

AS can be seen in [Figure 5,](#page-6-0) the regional precipitation-frequency curve predicted an AEP for the PMF of approximately 1E-6 or less frequent. The rainfall-runoff results differ significantly from the Bayesian fit using historical and paleoflood data. These two differing results can lead to very different investment decisions. The rainfall-runoff results suggest that the chance of exceeding 5,100 cms is de minimis, or practically zero. Whereas, the Bayesian fit with the temporal data suggests that there is a non-zero chance.



<span id="page-6-0"></span>*Figure 5. The expected 3-day basin average regional precipitation-frequency curve (shown in blue) and rainfallrunoff results from HEC-HMS (shown as colored diamonds) compared with the Bayesian fit of the LPIII distribution using systematic, historical, and paleoflood data at Lookout Point Dam.* 

The regional rainfall-frequency and modelled runoff results can be incorporated into the Bayesian analysis using by estimating the prior distribution of discharge for a specified flood quantile [(Coles & Tawn, 1996); (Viglione, Merz, Salinas, & Bloschl, 2013)]. For this study, a distribution of discharge was estimated for the 1:10,000 (1E-4) return period. This rare quantile was selected because in order to add information to the fit, it needed to be rarer than the paleoflood event, while not being so rare as to be overly influential.

The distribution of rainfall at the 1:10,000 (1E-4) return period was determined to be approximately normally distributed, The HEC-HMS rainfall-runoff results at all of the sites indicate that rainfall losses can vary significantly depending on the antecedent conditions and the spatial and temporal temperature pattern of the event. Also, there can be additional excess runoff due to initial SWE in the basin, or there can be significant losses due to precipitation turning to snow accumulation. Using the modelling results and expert judgment, the distribution of rainfall losses were considered to be triangularly distributed, with a lower bound of -2 inches to account for initial SWE, a mode of 2.5 inches, and an upper bound of 5 inches. This resulted in an estimated distribution of 3-day rainfall-runoff at the 1E-4 quantile that is log-normally distributed with a mean of 2,060 cms and a standard deviation of 570 cms, as shown in [Figure 6.](#page-7-0) The likelihood function for the Bayesian analysis is then constructed by first defining the prior distribution for the flood quantile:

$$
h(Q_P) = N(\mu_P, \sigma_P) \qquad \qquad \text{Equation 9}
$$

Where  $Q_p$  is the flood discharge for a specific annual exceedance probability P; and  $h(\cdot)$  is the pdf of the log-normally distribution variable  $Q_P$ .  $Q_P$  is determined from the inverse cdf  $X_F(\cdot)$  of the LPIII conditional on the parameters.

$$
Q_P = X_F(P|\theta) \qquad \qquad \text{Equation 10}
$$

The inverse cdf is then plugged into the pdf of the flood quantile to get:

$$
\pi(\theta) = h(X_F(P|\theta))
$$
 Equation 11



<span id="page-7-0"></span>*Figure 6. Distribution of 3-day rainfall-runoff at the 1:10,000 (1E-4) quantile, modeled with a log-Normal distribution with a mean µ = 2,900 cms and standard deviation σ = 570 cms. The 5th and 95th percentiles are marked with vertical black dashed lines.* 

During the Bayesian MCMC routine, the likelihood of the various components are multiplied by the causal prior  $\pi(\theta)$  to calculate a posterior distribution of the parameters, which is consistent with a reasonable range of  $O_p$ . The overall likelihood function is then constructed by putting together the components:

<span id="page-7-1"></span>
$$
L(D|\theta) = L_S(D|\theta) \cdot L_L(D|\theta) \cdot L_I(D|\theta) \cdot \pi(\theta) \qquad \qquad \text{Equation 12}
$$

This approach<sup>[5](#page-7-2)</sup> for causal information is consistent with the methods presented in *Viglione et al.* (2013) and *Skahill et al.* (2016). It can be seen in [Equation 12](#page-7-1) that the causal indicator  $\pi(\theta)$  acts similar to a penalty function, in that it rewards parameter sets that produce a reasonable likelihood of  $Q_p$  and discounts those that do not. All other things being equal, parameter sets that do not agree well with  $Q_P$  will have lower likelihoods than those that do.

The results of the Bayesian MCMC algorithm are shown in [Figure 7.](#page-8-0) The distribution of runoff at the 1:10,000 (1E-4) return period is represented by a green marker for the median and with error bars for the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the quantile distribution. The inclusion of rainfall-runoff data into the Bayesian analysis brings the volume-frequency curve into better agreement with the HEC-HMS results.

The combination of temporal, spatial and causal information expansion significantly increases the ERL beyond that of the temporal expansion alone. This is because the regional precipitation-frequency curves had ERLs that ranged between approximately 350 to 675 years, indicating high confidence in the regional quantile estimates.

[Figure 7](#page-8-1) shows a comparison of the posterior mode estimation (points), expected values (diamonds), and 90% credible bounds of Q1E-4 and the PMF return period for the three fitting scenarios. The 90% credible bounds of the Bayesian fit in the range of the 1:10,000 (1E-4) return period are much narrower as compared to the fit with only temporal information. The uncertainty in the PMF return period has been reduced as well. There is less than a half order of magnitude between the expected and most likely curves; the expected return period of the PMF is approximately 1:140,000 while the posterior mode predicts a PMF return period of approximately 1:400,000.

<span id="page-7-2"></span><sup>&</sup>lt;sup>5</sup> The method presented in *Viglione et al.* (2013) provides partial information on the priors. RMC-BestFit offers a more complete treatment of the priors following the procedures outlined in *Coles and Tawn* (1996). Both approaches produce comparable results when eliciting a prior for a single quantile.



#### Lookout Point 3-Day Volume-Frequency, with Historical, Paleoflood & Causal Information

<span id="page-8-0"></span>*Figure 7. Bayesian fit of the LPIII distribution using systematic, historical, paleoflood and causal information at Lookout Point Dam.* 



<span id="page-8-1"></span>*Figure 8. Posterior mode estimation (points), expected values (diamonds), and 90% credible bounds of Q1E-4 and the PMF return period.*

# **Comparison to EMA**

The currently accepted flood-frequency methodology in the United States, described in Bulletin 17C (U.S. Geological Survey, 2018), recommends fitting the LPIII distribution using the Expected Moments Algorithm (EMA), which is capable of incorporating historical and paleoflood information into flood frequency studies. While EMA is a major improvement over the Bulletin 17B guidelines (U.S. Geological Survey, 1982), it is still limited as compared to Bayesian approaches.

EMA was developed as an alternative to Maximum Likelihood Estimation (MLE) and the Bulletin 17B (U.S. Geological Survey, 1982) method for incorporating historical information in flood frequency studies (Cohn, Lane, & Baier, 1997). EMA was shown to achieve greater efficiency than the B17B adjustment for censored data, and nearly achieving the efficiency of MLE while avoiding some of the numerical complications. However, in the case when there is no censored data, EMA and B17B are identical and exhibit substantial bias as compared to MLE (Cohn, Lane, & Baier, 1997).

The Bayesian approach is closely related to the MLE method. The MLE method formulates a likelihood function using sample data,  $D = (X_1, ..., X_n)$ , and the parameter,  $\theta$ , of the frequency distribution, to seek the value of the parameters that maximize the likelihood function. The likelihood function gives the probability of the data, conditional on the distribution parameters (as shown i[n Equation 2\)](#page-1-1). The primary difference between the Bayesian approach and the MLE method is that the Bayesian analysis accounts for the frequency distribution parameter uncertainty, whereas the MLE method only provides the most likely parameter set.

[Figure 8](#page-9-0) shows a comparison between the Bayesian MCMC method and EMA. The comparison shows that both methods produce similar flood frequency curves for systematic, historical, and paleoflood data. There are minor trade-offs between mean, standard deviation, and skew between the two estimation methods, and the confidence intervals are slightly different. However, the differences are inconsequential, and the computed curves for each method are very consistent for rare return periods.

Both methods produce similar results given typical censored data; however, EMA is not capable of accounting for uncertainty in paleoflood ages and magnitudes, nor is it capable of incorporating the causal rainfall-runoff information in a formal, probabilistic manner. Whereas, the Bayesian estimation method is capable of incorporating and accounting for uncertainty in all available sources of hydrologic information, such as systematic flood records, historical floods, paleofloods, and regional rainfall-runoff results. Consequently, the Bayesian method is considered superior, and provides higher confidence in the fitted volume frequency curves and the resulting estimates for design floods with a specified return period.



#### Lookout Point 3-Day Volume-Frequency, with Historical & Paleoflood Data

<span id="page-9-0"></span>*Figure 9. Comparison of the Bayesian fit to the EMA fit using systematic, historical, and paleoflood data.*

# **Conclusions**

This paper considers a Bayesian flood frequency approach to estimate design flood values for use in Dam and Levee Safety modification studies. The Bayesian flood frequency approach is capable of incorporating all available sources of hydrologic information, such as paleofloods, regional rainfall-runoff results, and expert elicitation. All of this information can be combined into one flood frequency curve that can used to evaluate reservoir stage-frequency and the return periods of design floods in risk assessments and Dam and Levee safety modification studies.

The Lookout Point Dam case study illustrates by example how three pieces of information (temporal, spatial, and causal) can be combined with the systematic flood data in a Bayesian analysis. The ability of the Bayesian approach to use all pieces of information in conjunction is a major advantage over other methods, such as EMA, and provides much better estimates of design floods with specified return periods.

Before combining information, our estimate of the return period for the PMF at Lookout Point Dam could have been anywhere from 1E-4 to much less than 1E-8. This would render it nearly impossible to make effective design decisions. It is suggested that complementing systematic flood data by temporal, spatial, and causal information should become the standard procedure for estimating large return period floods.

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## **References**

- Beard, L. R. (1960, July). Probability Estimates Based on Small Normal-Distribution Samples. *Journal of Geophysical Research*.
- Bonneville Power Agency. (2017). *NRNI\_Flows\_1929-2008\_Corrected\_04-2017.csv*.
- Cohn, T. A., Lane, W. M., & Baier, W. G. (1997). An Algorithm for Computing Moments-Based Flood Quantile Estimates When Historical Flood Information is Available. *Water Resources Research*, 2089-2096.
- Coles, S. G., & Tawn, J. A. (1996). A Bayesian Analysis of Extreme Rainfall Data. *Journal of the Royal Statistical Society*, 463-478.
- Hirsch, R. M., & Stedinger, J. R. (1987). Plotting Positions for Historical Floods and Their Precision. *Water Resources Research*, 715-727.
- Jongejan, R. (2018). *Uncertainty in Hydrology: An evaluation of USACE methodology for estimating and portraying hydrologic uncertainty.*
- Kuczera, G. (1999). Comprehensive at-site flood frequency analysis using Monte Carlo Bayesian inference. *Water Resources Research*, 1551-1557.
- Kuczera, G., & Franks, S. (2019). *Australian Rainfall and Runoff - A Guide to Flood Estimation - Book 3: Peak Discharge Estimation.* Barton, Australia: Institution of Engineers.
- Merz, R., & Bloschl, G. (2008). Flood frequency hydrology: 1. Temporal, spatial, and causal expansion of information. *Water Resources Research*.
- Meylan, P. (2012). *Predictive Hydrology: A Frequency Analysis Approach.* Enfield, New Hampshire: Science Publishers. O'Connell, D. R., Ostenaa, D. A., Levish, D. R., & Klinger, R. E. (2002). Bayesian flood frequency analysis with
- paleohydrologic bound data. *Water Resources Research, 38*(5). Retrieved from doi:10.1029/2000WR000028 Perreault, L. (2000). *Bayesian retrospective analysis of a break in sequences of hydrologic random variables.* Paris:
- ENGREF. Reis, D. S., & Stedinger, J. R. (2005). Bayesian MCMC flood frequency analysis with historical information. *Journal of Hydrology*, 97-116.
- Skahill, B. E., Viglione, A., & Byrd, A. (2016). *ERDC/CH: CHETN-X-1 A Bayesian Analysis of the Flood Freqeuncy Concept.* U.S. Army Corps of Engineers.
- Stedinger, J. R. (1983). Design Events With Specified Flood Risk. *Water Resources Research*, 511-522.
- Stedinger, J. R., & Cohn, T. A. (1986). Flood frequency analysis with historical and paleoflood information. *Water Resources Research, 22*(5).
- ter Braak, C. J., & Vrugt, J. A. (2008). Differential Evolution Markov Chain with snooker updater and fewer chains. *Statistics and Computing*, 435-446.
- U.S. Army Corps of Engineers. (2016b). *Hydrologic Model Calibration and Validation Report.* Portland.
- U.S. Army Corps of Engineers. (2017). *RMC-TR-2017-05 Willamette Basin Regional 72-Hour Wintertime Precipitation Frequency Analysis.*
- U.S. Army Corps of Engineers. (2018a). *Lookout Point Paleohydrology Study.* Denver.
- U.S. Geological Survey. (1982). *Guidelines for Determining Flood Flow Frequency Bulletin 17B.*
- U.S. Geological Survey. (2018). *Guidelines for Determining Flood Flow Frequency Bulletin 17C.* https://doi.org/10.3133/tm4B5.
- Viglione, A., Merz, R., Salinas, J. L., & Bloschl, G. (2013). Flood frequency hydrology: 3. A Bayesian analysis. *Water Resources Research, 49*(2). Retrieved from doi:10.1029/2011WR010782